On the design of proportional integral observer for a rotary drilling system

Madjid KIDOUCHE* and Rami RIANE**
Applied Automation Laboratory, F.H.C., University of Boumerdès, Av. de l'indépendance, 35000 Boumerdès, Algeria
(*) kidouche_m@hotmail.com
(**) riane.rami@gmail.com

Abstract. Torsional drill-string vibrations, also known as ”stick-slip” oscillations appearing in oil-well drill-strings are a source of failures which reduce penetration rates and increase drilling operation costs. Some strategies based on drillers recommendations are evaluated in order to reduce stick-slip oscillations, the use of the angular velocity at the drill-string upper part, and the weight on the bit is shown to have a key effect in the reduction of drill-string torsional vibrations. In this paper we design a Proportional Integral Observer (PIO) to estimate the down hole speed and detect stick-slip vibrations.

Keywords: component; stick-slip oscillations, drilling system, dry friction, Proportional Input observer, Unknown Input observer

1 Introduction

The drilling technique used mostly in the oil industry, called rotary drilling, it is the creation of a bore hole by mean of a tool, called a bit. This technique relies on a mechanical system for energy transport from surface to the bit and hydraulic system for material transport from the bit to surface. The mechanical part is composed of a rotating bit to generate the bore-hole, a drill-string to rotate the bit, a rotary drive at surface to rotate the drill-string. The hydraulic part consists of mud (drilling fluid), pumps, and a transport channel[1]. When drilling, drill-string is exposed to vibrations, they are classified depending on direction they appear, we distinguish three main types of them: torsional, axial, and lateral (see[1][2]). These vibrations can exist separately or can be present together. Rotary drilling systems using drag bits, which consist of fixed blades or cutters mounted on the surface of a bit body, are known to exhibit, mainly, torsional vibrations, which may lead to torsional stick-slip, characterized by sticking phases with the bit stopping completely and slipping phases with the angular velocity of the tool increasing up to times of the angular velocity at the surface. These stick-slip oscillations decrease drilling efficiency, accelerate the wear of drag bits and may cause drill-string failure because of fatigue. In this paper we focus on modeling the behavior of the drill-string torsional vibration and designing an observer for the Bottom Hole Assembly (BHA) dynamic in

8^{th} *CHAOS Conference Proceedings, 26-29 May 2015, Henri Poincaré Institute, Paris France*

order to detect stick-slip vibrations. Most of the models describing stick-slip motion in drill-string consider this last as a torsional pendulum with different degrees of freedom [3][4]. A simple and reliable model can be obtained with two degrees of freedom as in [3][5][6][7]. An unknown input proportional integral observer is designed in order to estimate the unmeasured states and the unknown input.

2 Torsional model of a drill-string

A simple model of torsional drill-string vibrations is obtained by assuming that the drill-string behaves as a torsional pendulum, i.e. the drill pipes are represented as torsional spring, the drill collars behave as a rigid body and the top drive rotates at constant speed, (see Figure 2). In this study, It is supposed that no lateral or axial motions of the bit are present, and the only part of the drill-string interacting with the bore hole is the bit. This interaction is usually modeled by frictional forces (dry friction model) [3][1][7]. In this paper, we consider the interaction between the bit (drag bit) and the rock as a combination of two processes: cutting of the rock and frictional contact[8][9][5]. The corresponding equation of motion are written.

A. The Bottom-hole-assembly (BHA)

The bottom-hole-assembly dynamic is governed by the following equation:

$$J_b\ddot{\varphi}_b = k(\varphi_t - \varphi_b) - C_b\dot{\varphi}_b - Tob(\dot{\varphi}_b) \quad (1)$$

Let us set:

$$\phi = \varphi_t - \varphi_b\,,\ \text{And,}\quad \dot{\varphi}_b = \Omega_b$$

Then:

$$J_b\dot{\Omega}_b = k\,\phi - C_b\Omega_b - Tob(\Omega_b) \quad (2)$$

Where: (φ_b) (Ω_b) (J_b) (C_b) are respectively, the angular displacement, angular velocity, equivalent of mass moment of inertia, equivalent viscous damping coefficient at the bottom of the drill-string, and (φ_t) is the angular displacement at the top of the drill-string, (k) is the torsional stiffness coefficient, and *Tob* is a nonlinear function which will be referred to be the torque-on-bit.

B. The Drive system

The mechanical behavior of the drill sting at the surface is dominated by two components: a gearbox with combined gear ratio of (n:1), and an electric motor (here considered as a separately excited DC motor). The equations of this system are given as follow.

- Mechanical Equation:

$$J_t\dot{\Omega}_t = T - C_t\Omega_t - k\,\phi \quad (3)$$

Where (Ω_t) (J_t) (C_t) are respectively, the angular velocity, equivalent of mass moment of inertia, equivalent viscous damping coefficient at the top of the drill-string, T is the torque delivered by the motor to the system multiplied by the gearbox ratio (*n*). $T = n\,T_m$

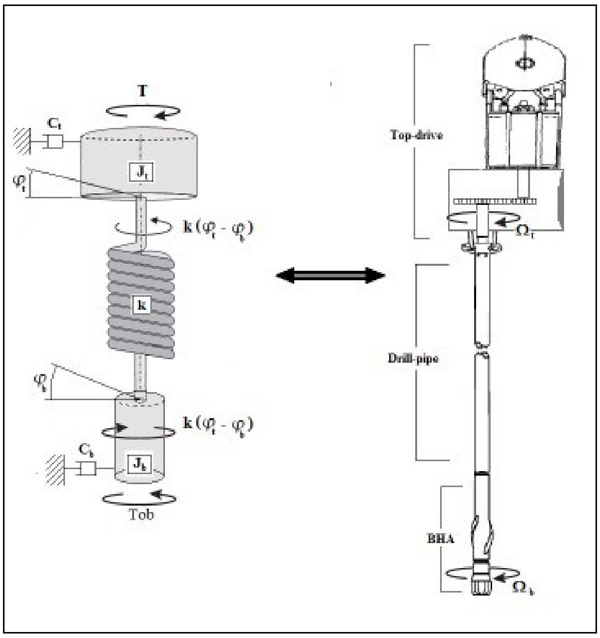

Figure 1. Mechanical model describing the torsional behavior of a generic drill-string.

- Electrical Equation:

$$v = l\frac{di}{dt} + r\,i + v_{cem} \qquad (4)$$

Where *(l), (r), (i)* and *(v)* are respectively defined as motor current, motor resistance, motor inductance and motor input voltage, *(v_{cem})* is the counter-electromotive force (back-emf), *(K)* is the motor constant multiplied by the gearbox ratio such as. $K = n\,K_m$

The counter-electromotive force, and the motor torque, are linearly related to the motor speed and the motor current , respectively.

$$v_{cem} = K\,\Omega_t \text{ , And, } T = K\,i$$

Finally

$$J_t\dot{\Omega}_t = K\,i - C_t\Omega_t - k\,\phi \qquad (5)$$

$$l\frac{di}{dt} = v - r\,i - K\,\Omega_t \qquad (6)$$

C. Bit-Rock interaction (Torque-On-Bit)

As we say before, the Toque-On-Bit is the result of, not only a frictional contact, but also a cutting process [10].

$$Tob = T_c + T_f \tag{7}$$

According to [10], the cutting torque resulting from the cutting process is given by

$$T_c = \frac{1}{2} R_{bit}^2 \, \varepsilon \, d \tag{8}$$

Where (ε) is the intrinsic specific energy (the amount of energy required to cut a unit volume of rock), (d) the depth of cut, and (R_{bit}) is the bit radius. Unlike [8] [5] where the frictional contact is modeled as static continuous model, here the frictional contact is modeled as a dynamic discontinuous dry friction contact. One of the difficulties with modeling friction is the complexity of the phenomenon at low velocity and, in particular, of the stick-slip process. In the slip phase, the macroscopic relative motion is null and friction appears as constraint maintaining the zero-velocity condition between the rubbing surfaces. One extensively-used model is the classical Coulomb model shown in Figure 3 , this model exhibits numerical problems in the vicinity of zero-velocity, therefor KARNOOP [11] introduce a zero velocity band in his model where a condition for switching from the stick to the slip motion is established , outside this band a standard Coulomb model is used. The Karnopp model is given by:

$$\mathrm{T}_f = \begin{cases} T_e & si \ \ |\Omega_b| < D_v \ \ et \ \ |T_e| < T_s \\ T_s \, sign(T_e) & si \ \ |\Omega_b| < D_v \ \ et \ \ |T_e| > T_s \\ T_d \, sign(\dot{\varphi}_b) & si \ \ |\Omega_b| \geqslant D_v \end{cases} \tag{9}$$

Where (T_e) is the applied external torque that must overcomes the static friction torques (T_s), (T_d) is the dynamic friction torque, and (D_v) is the zero-velocity band. With:

$$T_{s(d)} = \frac{1}{2} \ Wob \, g \ \ \mu_{s(d)} R_{bit} \tag{10}$$

Where ($\mu_{s(d)}$) is the static (dynamic) dry frictional coefficient, (Wob) is the Weight-On-Bit. Finally, the torsional model of the whole system is writing as:

$$\begin{cases} \dot{\phi} = \Omega_t - \Omega_b \\ \dot{\Omega}_t = \dfrac{-k}{J_t} \phi - \dfrac{C_t}{J_t} \Omega_t + \dfrac{n K}{J_t} i \\ \dot{\Omega}_b = \dfrac{k}{J_b} \phi - \dfrac{C_b}{J_b} \Omega_b - \dfrac{1}{J_b} Tob(\Omega_b, Wob) \\ \dfrac{di}{dt} = \dfrac{-n K}{l} \Omega_t - \dfrac{r}{l} i + \dfrac{1}{l} v \end{cases} \tag{11}$$

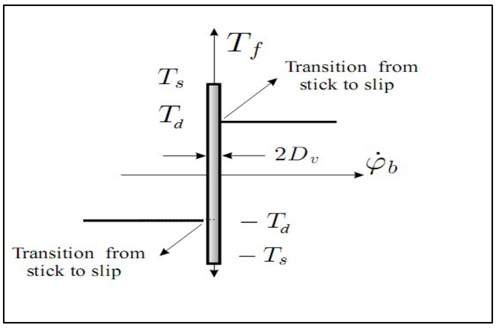

Figure 2. Models for the friction between the bit and the rock given by expression (9)

The overall system architecture was implemented as shown in Figure 4

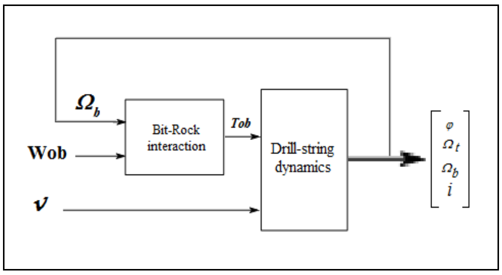

Figure 3. Block diagram of a torsional model of the drill-string

4 Observer design of the rotary drilling system

The proportional integral observer (PIO) is an observer in which an additional term, which is proportional to the integral of the output estimation error is added in order to achieve some desired robustness performance. Here we design a Proportional Integral Observer (PIO) for unknown input (Tob) which estimates both the state and the bounded nonlinear unknown input as shown in figure (5), (See [12][13]).

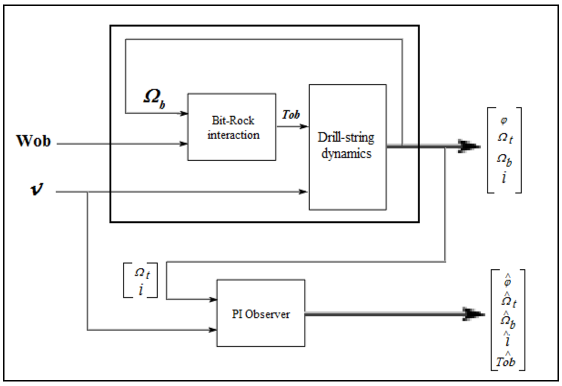

Figure 4. Block diagram of a PI Observer for unknown input.

A continuous-time linear state space model of the drill-string torsional behavior can be derived from (11) as:

$$\begin{cases} \dot{X} = A\,X + B\,u + E\,d \\ Y = C\,X \end{cases} \tag{12}$$

Where $X = (\Phi\ \Omega_t\ \Omega_b\ i\ )^t$ is the state vector, $Y = (\Omega_t\ \ i\ )^t$ is the output vector, $(u = v)$ is the known input (voltage), ($d = Tob$) is an unknown input (Torque-On-Bit). A, B, C, and E are known matrices with appropriate dimensions.

$$A = \begin{pmatrix} 0 & 1 & -1 & 0 \\ \frac{-k}{J_t} & \frac{-C_t}{J_t} & 0 & \frac{K}{J_t} \\ \frac{k}{J_b} & 0 & \frac{-C_b}{J_b} & 0 \\ 0 & \frac{-K}{l} & 0 & \frac{-r}{l} \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{l} \end{pmatrix} \qquad E = \begin{pmatrix} 0 \\ 0 \\ \frac{-1}{J_b} \\ 0 \end{pmatrix} \qquad C = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The PI observer (14) is synthesized on the basis of the following augmented system

$$\begin{cases} \dot{Z} = A_z Z + B_z u \\ Y = C_z Z \end{cases} \tag{13}$$

Where:

$$Z = \begin{pmatrix} X \\ d \end{pmatrix} \qquad A_z = \begin{pmatrix} A & E \\ 0 & 0 \end{pmatrix} \qquad B_z = \begin{pmatrix} B \\ 0 \end{pmatrix} \qquad C_z = \begin{pmatrix} C & 0 \end{pmatrix}$$

Since the pair (A_z, C_z) is observable, a PI observer for the system (12) can be defined as follows:

$$\begin{cases} \dot{\hat{X}} = A\,\hat{X} + B\,u + E\,\hat{d} + K_P (Y - C\,\hat{X}) \\ \dot{\hat{d}} = K_I (Y - C\,\hat{X}) \end{cases} \tag{14}$$

Where (K_P) and (K_I) are proportional and integral gains respectively, (14) is written in the following augmented form.

$$\begin{cases} \dot{\hat{Z}} = A_z \hat{Z} + B_z u + K_z (Y - C_z \hat{Z}) \\ \hat{Y} = C_z \hat{Z} \end{cases} \tag{15}$$

Where: $\hat{Z} = \begin{pmatrix} \hat{X} & \hat{d} \end{pmatrix}^t$; $K_z = \begin{pmatrix} K_P & K_I \end{pmatrix}^t$

The state estimation error of the augmented system is

$$e = Z - \hat{Z}$$

So:

$$\dot{e} = \dot{Z} - \dot{\hat{Z}} = (A_z - K_z C_z) e \tag{16}$$

Estimation errors converge asymptotically to zero if the matrix $(A_O=A_z - K_z C_z)$ is Hurwitz. Since the pair (A_z, C_z) is observable, the gain (K_z) can be calculated by pole placement.

5 Simulation results

Simulations have been performed to investigate the efficiency level of the proposed model and observer for different scenarios of wight-on-bit and motor voltage, these simulations are performed with Lab-View. The numerical values used in these simulations are listed in table 1 and table 2 (see Appendix).

TABLE 1. NUMERICAL VALUES OF THE DRILLING SYSTEM

Parameter	*Description*	*Value*	*Unit*
J_t	Equivalent of mass moment of inertia at the top of the drill-string	999.35	kgm^2
J_b	Equivalent of mass moment of inertia at the bottom of the drill-string	127.27	kgm^2
C_t	Equivalent viscous damping coefficient at the top of the drill-string	51.38	*Nms/rad*
C_b	Equivalent viscous damping coefficient at the bottom of the drill-string	39.79	*Nms/rad*
k	Drill-string stiffness	481.29	*Nm/rad*
r	Motor resistance	0.01	Ω
l	Motor inductance	0.005	*H*
K_m	Motor constant	6	*Nm/A*
n	Gearbox ratio	7.20	-

TABLE 2. NUMERICAL VALUES OF THE BIT-ROCK INTERACTION.

Parameter	*Description*	*Value*	*Unit*
e	Intrinsic specific energy of the rock	130	MJ/m^3
d	Depth of cut	4	*mm/rev*
μ_s	Static dry frictional coefficient	0,6	-
μ_d	Dynamic dry frictional coefficient	0,4	-
R_{bit}	Bit radius	0.10	*m*

First, to approve the model (11), we test it on the basis of drillers observations and recommendations. In practice, to avoid stick-slip vibrations on the field, the driller operator typically controls the surface-controlled drilling parameters, such as the weight on the bit, the speed at the surface and the viscosity of the drilling fluid to limit or eliminate stick-slip vibrations.

- Manipulation of the weight on the bit and the speed at the surface

From field data experience, it is concluded that the reduction of the weight on the bit and/or the augmentation of the speed at the surface can limit the severity of stick-slip vibrations, we can see these facts by simulating model (11) as shown in figures (6,7).

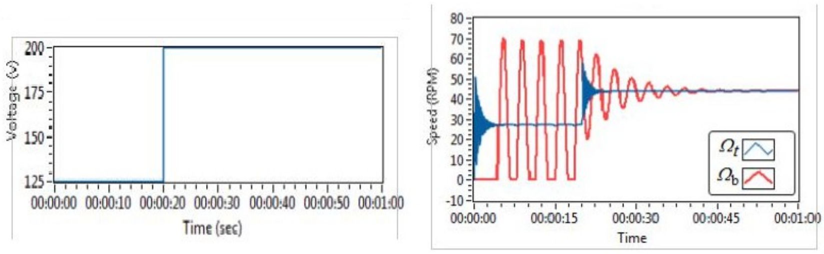

Figure 5. Angular velocity of the BHA (Ω_b) and the speed at the surface (Ω_t) under a constant weight-on-bit (Wob=15T) and augmentation of DC motor voltage from (v=125V) to (v=200V) at (t=20s)

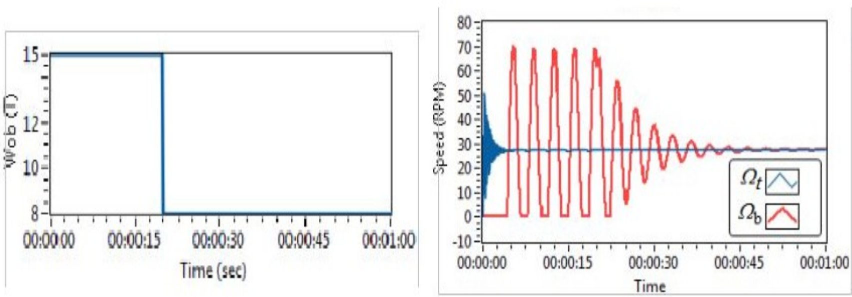

Figure 6. Angular velocity of the BHA (Ω_b) and the speed at the surface (Ω_t) under a constant DC motor voltage (v=125V) and decreasing weight-on-bit from (Wob=15T) to (Wob=8T) at (t=20s)

- Manipulation of the viscosity of the drilling fluid

Another strategy for reducing stick-slip oscillations at the BHA is by increasing the damping at the down end of the drill-string, this can be done by modifying the drilling fluid characteristics, we can also see that by simulating model (11) with increasing equivalent damping coefficients C_b and C_t as shown in figure(8).

As the above simulations reflect experimental results, the model (11) is validated.

Second, to estimate the speed of BHA and the torque on bit, eigenvalues of the observer are located at (-10 -50 -70 -100 -130), therefore the gain is calculated by “poleplace” commande of LabView MathScript node. Figures bellow (10,11) show the performance of the PIO (19) for the wight on bit and motor voltage inputs shown in figure (9).

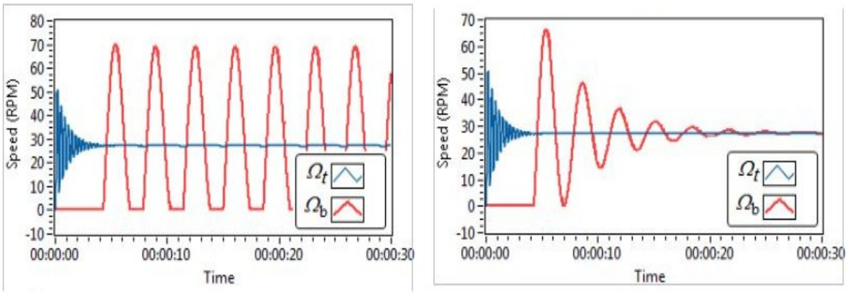

Figure 7. Angular velocity of the BHA (Ω_b) and the speed at the surface (Ω_t) under (Wob=15T) and (v=125V). Top : with (Ct=51.38 *Nms/rad* , Cb=39.79 *Nms/rad*), Bottom: with (Ct=63,54 *Nms/rad* , Cb= 56,85 *Nms/rad*)

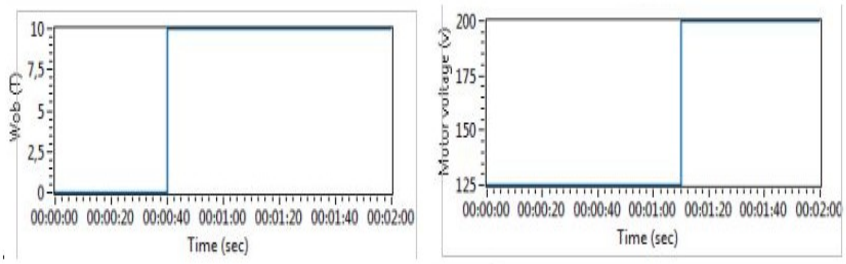

Figure 8. weight-on-bit and Motor voltage

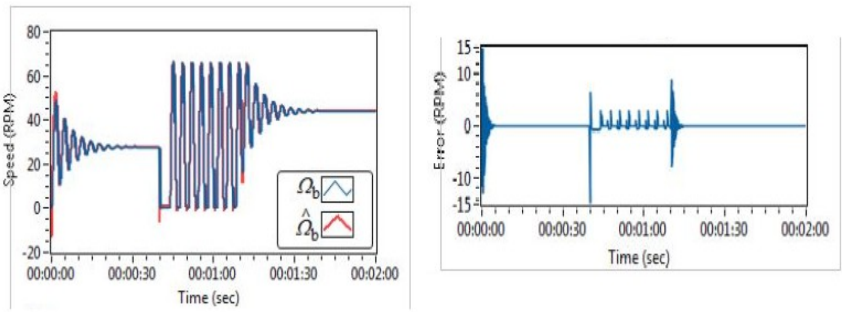

Figure 9. BHA speed and BHA speed estimation (on the left). estimation error of the speed at the surface (on the right)

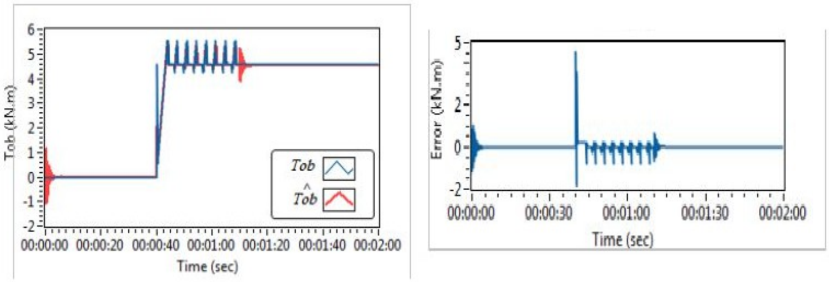

Figure 10. Torque-on-bit and Torque-on-bit estimation (on the left). estimation error of Tob (on the right)

Because of the weight-on-bit is usually perturbed due to axial vibrations, PIO (19) is tested under a perturbed Wob which causes a perturbation on torque-on-bit (unknown input) as shown in figure (12). To enhance our results, eigenvalues of the observer are placed at (-10 -100 -110 -120 -130).

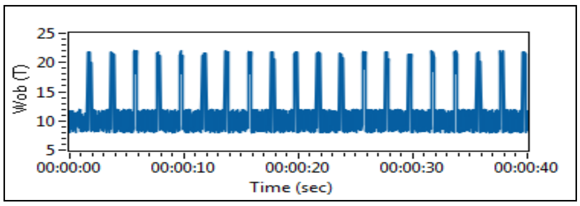

Figure 11. weight-on-bit

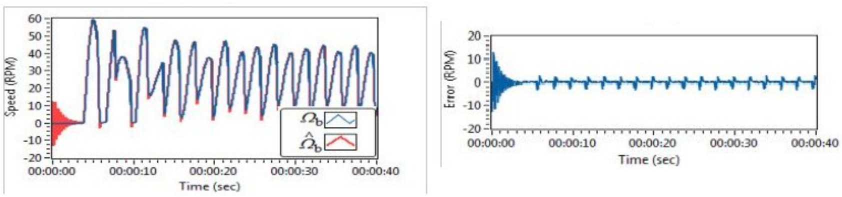

Figure 12. BHA speed and BHA speed estimation (on the left). Estimation error of the speed at the surface, under Wob shown in figure 12 and motor voltage =125V (on the right)

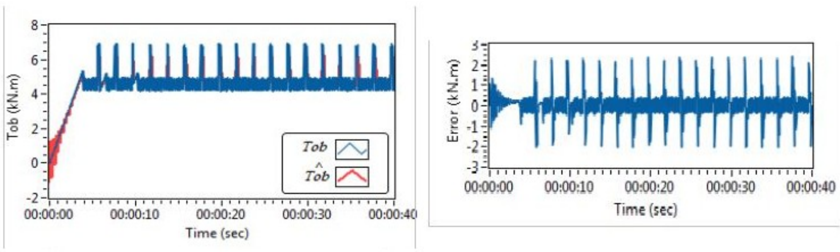

Figure 13. Torque-on-bit and Torque-on-bit estimation (on the left). Estimation error of Tob (on the right)

6 Conclusion

This paper has studied stick-slip vibrations in oil-well drill-string. A models for describing the torsional drill-string behavior have been given. We presented also a model for the rock-bit interaction, the modeling of exact friction characteristic is not an easy problem, because the friction characteristic can be changed easily due to the environment's changes. A PI observer has been developed to estimate the speed of the BHA, in which the unknown disturbance (Torque-on-bit) has been also estimated, this estimation can be used to compensate the torque-on-bit in vibration suppression of a drill-string.

Appendix

Numerical values listed in table 1 are calculated by the following equations:

- Mass moment of inertia

$$J_t = J_m + \frac{1}{3} J_p \quad \text{and} \quad J_b = J_c + \frac{1}{3} J_p$$

(J_m, J_c, J_p) : mass moment of inertia of the motor, the drill-collar, the drill-pipe respectively. ($J_{t(b)}$) equivalent mass moment of inertia at the top of the drill-string (at the bottom of the drill-string).

Where: $J_p(L_p) = \frac{1}{2} \rho\pi (R_p^4 - r_p^4) L_p$ and $J_c(L_c) = \frac{1}{2} \rho\pi (R_c^4 - r_c^4) L_c$

$R_{p(c)}$ is the outer radius of the drill-pipe (drill-collar), $r_{p(c)}$ is the inner radius of the drill-pipe (drill-collar), $L_{p(c)}$ is the length of the drill-pipe (drill-collar), ρ is the density of steel.

- The torsional stiffness coefficient $k(L_p) = \frac{G I}{L_p}$

Where: $I = \frac{\pi}{2} (R_p^4 - r_p^4)$

(k) is the torsional stiffness coefficient, *(G)* shear modulus of steel, *(I)* is the polar moment of inertia.

- The viscous damping coefficients

$$C_t = C_m + \frac{1}{2} C_p \quad \text{and} \quad C_b = C_c + \frac{1}{2} C_p$$

$C_{p(c)}$ viscous damping coefficient of the drill-pipe (drill-collar), (C_m) viscous damping coefficient of the motor.

Where: $$C_{p(c)}=120\,\eta\,\frac{R_h^2 R_{p(c)}^2}{R_h^2-R_{p(c)}^2}\,L_{p(c)}$$

(R_h) is the radius of the borehole, (η) is the dynamic viscosity

References

[1] Aurelian Iamandei and Gheorghe Miloiu, "Motor Drives of Modern Drillingand Servicing Rigs for Oil and Gas Wells," Springer Science+Business Media Dordrecht, 2013.

[2] Bart Besselink, Nathan van de Wouw, Henk Nijmeijer, "Model-based analysis and control of axial and torsional stick-sliposcillations in drilling systems," IEEE International Conference on Control Applications (CCA), 2011 .

[3] Damien Koenig and Saïd Mammar, "Design of Proportional-Integral Observer for UnknownInput Descriptor Systems," IEEE Transactions on automatic control, vol. 47, no. 12, pp. 2057-2062, December 2002.

[4] Dean Karnopp, "Computer simulation of stick-slip friction in mechanical dynamic systems, asme journal of dynamics systems," ASME Journal of Dynamics Systems, Measurement, and Control,Vol. 107, No. 1, pp. 100-103, 1985.

[5] E. Detournay and P. Defourny, "A phenomenological model for thedrilling action of drag bits," Int. J. Rock Mech. Min. Sci. & Geomech. Abstr. Vol. 29. No. 1, pp. 13-23, 1992 .

[6] E.M. Navarro-López and R. Suárez, "Modelling and Analysis of Stick-slip Behaviour in a Drill-string under Dry Friction," In the congress of the Mexican Association of Automatic Control, 2004.

[7] Emmanuel Detournaya, Thomas Richard, Mike Shepherd, "Drilling response of drag bits: Theory and experiment," International Journal of Rock Mechanics & Mining Sciences, vol. 45, pp. 1347–1360, 2008..

[8] F. Abdulgalil and H. Siguerdidjane , "Nonlinear friction compensation design for suppressing stick-slip oscillations in oil-well drill-strings," In 7th IFAC DYCOPS, Massachusets, USA, 2004.

[9] Johan Dirk Jansen, "Nonlinear dynamics of oilwell drill-strings," Doctoral thesis, Delft University, 1993.

[10] Ju Zhang, "Evaluation of observer structures with applicationto fault detection," Electrical and Computer Engineering Master'sThesis, Northeastern University, 2009.

[11] Lin Li, Qi-zhi Zhang and Nurzat Rasol , "Time-Varying Sliding Mode Adaptive Controlfor Rotary Drilling System," Journal of computers, vol. 6, no. 3, 2011.

[12] M. Krøtøy JohannessenTorgeir Myrvold and T. Myrvold, "Stick-Slip prevention of drill strings using nonlinear model reduction and nonlinear model predictive control," Master of Science in Engineering Cybernetics, Norwegian University of Science and Technology, 2010.

[13] Martin T. Bayliss, Neilkunal Panchal, and J. F. Whidborne , "Rotary Steerable Directional Drilling Stick-Slip Mitigation Control," In the proceedings of the 2012 IFAC Workshop on Automatic Control in Offshore Oil and Gas Production, Norwegian University of Science and Technology, Trondheim,Norway, May 31 - June 1, 2012.

[14] Nenad Mihajlović, "Torsional and Lateral Vibrations in Flexible Rotor Systems with Friction," Doctoral thesis, Technical University of Eindhoven, 2005.

[15] Thomas Richard, Christophe Germay, Emmanuel Detournay , "Self-excited stick–slip oscillations of drill bits," C. R. Mecanique, vol. 332, pp. 619–626, 2004.