Decentralized Control Design Using a New Approach of Longitudinal Multi-overlapping Decomposition Strategy: Application to Web Winding System

M. Kidouche [*], M. Z. Doghmane [+], H. Habbi [*]

[*] *Laboratory of Applied Automatic LAA*
Avenue Elistiklal Boumerdes 35000, Algeria
kidouche_m@hotmail.com
[+] *faculty of hydrocarbons and chemistry*
University M'hamed Bougara, Boumerdes, Algeria
doghmane_m@yahoo.com, habbi_hacéne@hotmail.com

***Abstract*— Industrial systems are generally characterized by their complication which is essentially caused by the large number of variables exist in such systems, and also the overlapping structure of information coming to or going from it. This complication can afford loss of some important parts of information, in addition to making study of these systems, if not impossible, at least very hard for researchers. Many researchers discuss the development of techniques to profit the structure of the system under study in order to reduce control complications. The aim of this paper is to use a proposed longitudinal approach to multi-overlapping controller design of large scale industrial systems; this approach permits us to show the advantages of multi-overlapping decomposition strategy and its application in order to have a more robust controller. Thus, it leads to more computational efficient design strategy that is well suited for large scale applications. A multi-overlapping web winding system with longitudinal structure is presented to illustrate the effectiveness of the proposed design.**

***Keywords*— Overlapping Structure; Multi-overlapping Controller Design; Large Scale Industrial Systems; Multi-overlapping Decomposition Strategy; Web Winding System; Longitudinal Structure**

I. INTRODUCTION

Since most of industrial systems are found very complicated, study of large scale systems dynamic becomes more interesting to researchers; the complications come first from the mathematical model, then its analysis. Thus, many strategies have been developed for dealing with these difficulties such as overlapping decomposition; it is based on the idea of subdividing the whole system into smaller subsystems less difficult, in study, than the original one. Nevertheless, this technique requires some conditions to apply, the challenge in such study is to find real system comply with these conditions. Moreover, large scale industrial systems are really difficult to be controlled globally without the simultaneous development of numerically efficient algorithms for convex optimization. Web winding systems are used for transporting paper, metal, polymers or textile and they are very common in industry [1]. The main goal of such systems is to increase as much as possible the web transport velocity while controlling the tensions of the web. Describing the behavior of such systems leads to high dimensional mathematical models; furthermore, the analysis and design of such large scale models becomes very complex because the amount of computational effort required to analyze dynamic process grows much faster so that the control tasks cannot be simply solved [2].

The complexity of study necessitates new design methodologies and ideas for decomposing and dividing the analysis and control problem of the overall system into smaller and easier to deal with sub-problems, so that, solutions of the sub-problems can be combined with interconnection constraints to find global solution of the whole system [3]. Several works about controlling the web winding system can be found in ([20]). Multivariable control strategies have been proposed for industrial metal transport systems [6], and H_∞ robust control is presented for decoupling web tensions and velocities. A few researchers have examined web winding systems; Ikeda et al. [7] proposed robust controllers that address nominal radii variations. Li et al. [1] proposed robust decentralized multi-overlapping controller with radial structure assuming that the frequency of the disturbance is constant and known.

Industrial windings are generally large scale systems and they are good applications for the recent improvements in decentralized control theories [5]. Moreover, traditional control strategies based on PID do not achieve good decoupling (especially for flexible webs). Furthermore, robust multivariable controllers, recently proposed for this application are only powerful for reduced size systems. Decentralized adaptive fuzzy control strategy is proposed in [9] for large scale nonlinear system using state and output feedback controller, the decentralized control schemes via state and output feedback guarantee the stability of the closed-loop large-scale systems. The effectiveness of these approaches is demonstrated through simulation results of a platoon of vehicles within an automated highway system [10], it is an innovative strategy for winding systems. It is based on multivariable control strategies without the inconvenient of having too many inputs and outputs for one controller [10].

Our objective is to develop an optimization algorithm for multi-overlapping decomposition with longitudinal structure, it allows us to decompose the complex systems into lower interconnected subsystems which can be stabilized locally, then conclude about global stability of the whole system.

The paper is organized as follows; in the second section the mathematical model of web winding system is presented based on previous works ([14], [20]), in the third section, multi-overlapping strategy is detailed in order to introduce the longitudinal structure of the system which is showed in the fourth section, then in section five decentralized optimal output feedback control algorithm is proposed according to the problem's frame of web winding and its solution. Finally, in the last section, interesting simulation results are shown for the proposed approach applied to five-motor web winding system.

II. Web Winding System Description

The system under consideration is quite common in industry; it is composed three motors: unwinding motor, traction motor and winding motor.

A. Winding Process

The system under study exhibits the inherent difficulties of elastic web winding systems ([14], [20]).

- The inputs: control signals T_u^*, V_3^* and T_w^*
- The outputs: unwinding web tension T_u, Traction motor's linear velocity V_3 ,Winding web tension Tw
- The web velocity is imposed by the traction motor and the web tension is controlled by the unwinding and winding motors.

The mathematical model of five motors web winding system can be described as follows [20]

First, the value of the web speed is bounded by two values $V_{\max}, V_{\min}$. Let us define ε as follows

$$\varepsilon = \frac{L_s - L_0}{L_0} = \frac{\Delta L}{L_0} \tag{1}$$

ε : is the strain of the web.

L : is the length of the web; L_s is the stretched length; L_0 is the un-stretched length.

B. Expression of Tensions

- **Hooke's law**

$$T = ES\varepsilon = ES\frac{L_s - L_0}{L_0} \tag{2}$$

Where E : is the Young's modulus of the web's elasticity.

S : The cross section area of the web span.

- **Law of mass' conservation**

The mass of the web remains constant between the state without stress and the state under stress

$$\rho_0 L\omega_0 h_0 = \rho_s L\omega_s h_s \Rightarrow \rho_0 S_0 = \rho_s S_s (1+\varepsilon)$$

$$\frac{m_s}{m_0} = \frac{\rho_s S_s}{\rho_0 S_0} = \frac{1}{1+\varepsilon} \tag{3}$$

where ρ : is the density of the span; ρ_s under stretched, ρ_0 before stretched.

m : is the mass of the web; m_s under stretched, m_0 before stretched; we assume that $S_s \cong S_0$, so that eq (3) becomes

$$\frac{\rho_s}{\rho_0} = \frac{1}{1+\varepsilon} \tag{4}$$

- **Law of continuity**

$$\frac{\delta\rho}{\delta t} + \frac{\delta(\rho V)}{\delta t} = 0 \tag{5}$$

$$dm = \rho_s S_s = \rho_0 S_0 \Rightarrow \frac{dm}{dt} = \frac{d}{dx}\left(\int_{x_1}^{x_2} \rho(x,t) S(x,t) dx\right)$$

$$= \rho_2(t) S_2(t) v_2(t) - \rho_1(t) S_1(t) v_1(t) \tag{6}$$

Assumptions

$$\begin{cases} \varepsilon_1(x,t) = \varepsilon_1(t) \Rightarrow \rho(x,t) = \rho_1(t) \\ S(x,t) = S_1(t) \end{cases} \tag{7}$$

From equations (4) and (6), we have

$$L\frac{d}{dx}\left[\frac{1}{1+\varepsilon_1(t)}\right] = \frac{v_2(t)}{1+\varepsilon_2(t)} - \frac{v_1(t)}{1+\varepsilon_1(t)} \tag{8}$$

Now by approximating

$$\frac{1}{1+\varepsilon} \cong 1-\varepsilon \Rightarrow \dot{\varepsilon}_1(t) = \frac{1}{L}\left[\varepsilon_2(t) v_2(t) - \varepsilon_1(t) v_1(t) + v_1(t) - v_2(t)\right]$$

$$\Rightarrow \dot{T}_1 = \frac{1}{L}\left[-v_1 T_1 + v_2 T_2 + SE(v_1 - v_2)\right] \tag{9}$$

For the i^{th} spam

$$L_i \dot{T}_i = ES(v_i - v_{i-1}) + T_{i-1} v_{i-1} - T_i (2v_{i-1} - v_i) \tag{10}$$

C. Expression of the Velocity

The velocity of the web span is equal to the tangential velocity of the roller; which means there is no sliding mode between the roller and web; it means that

$$\frac{d}{dt}(J_i \Omega_i) = R_i (T_{i+1} - T_i) + K_i U_i + C_f \tag{11}$$

where $\Omega_i = \frac{v_i}{R_i}$: is the angular velocity of the roller i .

$K_i U_i$: is the input torque.

C_f : is the torque resulted from the total friction coefficient.

J_i : is the moment of inertia of roller i .

R_i : is the radius of the roller i .

The variation of R_i is given by the following equation

$$\frac{dR_i(t)}{dt} = -\frac{hv_i(t)}{2\pi R_i(t)} \tag{12}$$

with h : is the thickness of the web.

The moments of inertia J_i can be calculated as follows

$$J_i(t) = J_{ai} + \frac{\pi\rho L_a}{2}(R_i^4(t) - R_{ai}^4) \tag{13}$$

L_a : is width of the web.

J_{ai}, R_{ai} : are the moment of inertia and radius of the motor i , and

$$F_i = \beta_{fi}\Omega_i \tag{14}$$

with $\Omega_i = \frac{v_i}{R_i(t)}$

where β_{fi} is the viscosity friction constant of the motor i .

We linearized these equations around the nominal velocity and tension (v_0, T_0), so that, Equation (10) becomes

$$L_i\dot{T}_i = (ES + T_0)(v_i - v_{i-1}) + v_0(T_{i-1} - T_i) \tag{15}$$

The state space representation of this linearized system is as follows

$$\begin{cases} E(t)\dot{X} = A(t)X + B(t)U \\ Y = CX + DU \end{cases} \tag{16}$$

where

$$\begin{cases} X^T = (v_1\ T_1\ v_2\ T_2\ v_3\ T_3\ v_4\ T_4\ v_5) \\ U^T = (U_u\ U_v\ U_w) \\ Y^T = (T_u\ v_3\ T_w) \end{cases} \tag{17}$$

$A(t), B(t), C(t)$ an d$E(t)$ are time varying matrices where

$$R_w(t) = \sqrt{R_u^2(0) + R_w^2(0) - R_u^2(t)} \tag{18}$$

The system described in eq (16) is time varying [14].

D. Assumption

Because the parameters of the system described above are varying very slowly with time, we can consider it constants in time interval $\Delta t = [t_0 \ \ t_{\text{simulation}}]$, so that the system is LTI in that interval. So, for each time's interval of parameters, we can construct an LTI system as follows

$$\begin{cases} E(t_i)\dot{X} = A(t_i)X + B(t_i)u \\ Y = CX + Du \end{cases}, i = 0,...,4 \tag{19}$$

For $D = 0$, eq (19) can be written as [13]

$$\begin{cases} \dot{X} = E(t_i)^{-1}A(t_i)X + E(t_i)^{-1}B(t_i)u \\ Y = CX \end{cases} \tag{20}$$

where

$$\begin{cases} E(t_i)^{-1}A(t_i) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \\ E(t_i)^{-1}B(t_i) = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix}, C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \end{cases} \tag{21}$$

Applying expansion-contraction principle, we can decompose system (20) using overlapping technique [8], as it is explained in the section three.

III. Multi-Overlapping Decomposition Strategy

Decomposition strategies of systems with multi-overlapping structure are effective techniques to solve problems in many domains such as applied mathematics, automated highway systems, electric power systems, and mechanical structures. The decomposition strategy is essentially based on expanding the state space representation, so that the overlapping subsystems appear as disjoint, applying standard methods for decentralized control design and contracting the obtained controller for implementation [3]. In this section, we consider a multi-overlapping structure with longitudinal topology. Basically, the overlapping parts are shared by multiple subsystems and decomposition based on a proper definition of expansion-contraction transformation matrices including convenient permutational transformations. Consider the system given in (20); this system can be rewritten as

$$S_i : \begin{cases} \dot{x}_i = A_{ii}x_i + B_{ii}u_i \\ y_i = C_{ii}x_i \end{cases} \quad i = 1,2,.....,N \tag{22}$$

The matrix A can be either in longitudinal, radial or loop structure; in this paper, we consider a longitudinal structure of the matrix A .

The main goal of expansion-contraction principle is to obtain an expanded structure in which subsystems appear as disjoint, to do this, we have to follow two steps.

In the first step, a transformation with full rank is applied to the overlapped system in order to separate the overlapping blocks and also to eliminate some interconnection terms by an appropriate choice of the complementary matrices. In the second step, permutational transformations and complementary terms are added in order to achieve the desired structure.

First, we choose the transformation matrices as

$$\begin{cases} T = blockdiag([I_{11}, I_{11}]^T, ..., [I_{NN}, I_{NN}]^T) \\ N = 0.5blockdiag([I_{11}, I_{11}], ..., [I_{NN}, I_{NN}]) \end{cases} \tag{23}$$

The complementary matrix E_A is chosen as

$$E_A = 0.5\times\begin{bmatrix} A_{11} & -A_{11} & A_{12} & -A_{12} & \cdots & -A_{1N} & A_{1N} \\ -A_{11} & A_{11} & A_{12} & -A_{12} & \cdots & -A_{1N} & A_{1N} \\ -A_{21} & A_{21} & A_{22} & -A_{22} & \cdots & 0 & 0 \\ -A_{21} & A_{21} & -A_{22} & A_{22} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ A_{N1} & -A_{N1} & 0 & 0 & \cdots & A_{NN} & -A_{NN} \\ A_{N1} & -A_{N1} & 0 & 0 & \cdots & -A_{NN} & A_{NN} \end{bmatrix} \quad (24)$$

So that, the original matrix is

$$A_e = \begin{bmatrix} A_{11} & 0 & A_{12} & 0 & \cdots & 0 & A_{1N} \\ 0 & A_{11} & A_{12} & 0 & \cdots & 0 & A_{1N} \\ 0 & A_{21} & A_{22} & 0 & \cdots & 0 & 0 \\ 0 & A_{21} & 0 & A_{22} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ A_{N1} & 0 & 0 & 0 & \cdots & A_{NN} & 0 \\ A_{N1} & 0 & 0 & 0 & \cdots & 0 & A_{NN} \end{bmatrix} \quad (25)$$

Secondly, we use the permutation matrix

$$T_A = T_{A12}T_{A23}...T_{A(2N-1)(2N)} = \prod_{k=1}^{2N-1} T_{Ak(k+1)} = \begin{bmatrix} 0 & \cdots & 0 & I_{11} \\ I_{11} & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & I_{NN} & 0 \end{bmatrix} \quad (26)$$

Resulting in the expanded system to be

$$\tilde{A}_e = T^I A_e T = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{12} & 0 \\ A_{21} & A_{22} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & A_{NN} & A_{N1} \\ 0 & A_{12} & \cdots & A_{1N} & A_{11} \end{bmatrix} \quad (27)$$

The expansion theorem can be used for designing decentralized control for a system with multi overlapping information structure constrains [16]. To stabilize each subsystem together with the whole interconnected system, we use the following local control laws within subsystems S_{ei}

$$\begin{bmatrix} u_i \\ u_j \end{bmatrix} = -\begin{bmatrix} K_{ii} & K_{ij} \\ K_{ji} & K_{jj} \end{bmatrix}\begin{bmatrix} x_i \\ x_j \end{bmatrix} \quad (28)$$

The decentralized control gain matrix for the system S_e is then given by

$$K_e = blockdiag(\cdots, \begin{bmatrix} K_{ii} & K_{ij} \\ K_{ji} & K_{jj} \end{bmatrix}, \cdots) \quad (29)$$

and the designed expanded controller is ,now, contractible [1].

IV. Longitudinal Structure of W.W.S

The mathematical model of web winding system including the derivation of all its variables is used in the simulation process [2]. Consider the i_{th} span in W.W system (Fig.2) where

- T_i, V_i : Tension, and velocity motor input.
- v_{i0} : Steady state web velocity; L : length between two driven rollers.
- S, E : Cross sectional area and Young's modulus of the web material.
- R_i, J_i, C_{emi} Radius, inertia, and motor torque constants.

A linearized model of the tension and the velocity variations can be given as

$$\begin{cases} \dot{T}_i = \dfrac{ES}{L}\left[-v_{i-1}\left(ES + T_{i-1}\right) + v_i\left(ES - T_i\right)\right] \\ \dot{v}_i = \dfrac{R_i}{J_i}\left[R_i\left(T_{i+1} - T_i\right) + Cem_i - f_i\left(t\right)\dfrac{v_i}{R_i}\right] \end{cases} \quad (30)$$

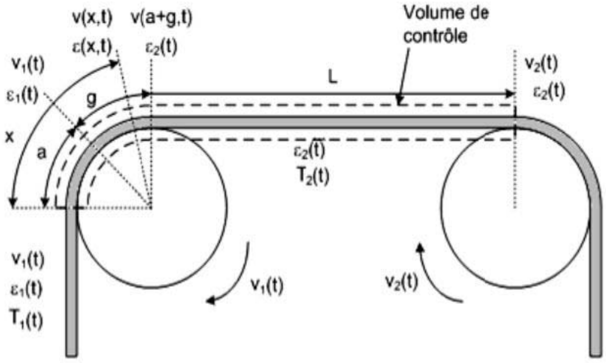

Fig.1. The i_{th} span in web winding system

The control design given for two processing spans can be generalized for all of the web winding system spans by considering that the radius (R_i), inertia (J_i), friction coefficient (B_{fi}) and motor torque constant (K_{mi}) are identical for all the driven rollers. The desired controller is developed to supply a master speed for the roller and to soothe tension fluctuations [14].

The state-space representation of the tensions and velocity is given as

$$\begin{cases} \dot{x}_p = A_n x_p + B_{n1}d + B_{n2}u \\ y = C_n x_p \end{cases} \quad (31)$$

where p stands for plant and n represents nominal.
For the first single span, the state space vectors and matrices are

$x_p = \begin{bmatrix} T_2 & V_1 & V_2 \end{bmatrix}^T$, $u = \begin{bmatrix} I_1 & I_2 \end{bmatrix}^T$, $y = \begin{bmatrix} T_2 & V_1 \end{bmatrix}^T$, $d = \begin{bmatrix} T_1 & T_3 \end{bmatrix}^T$

V. Decentralized Controller Design

A. Problem frame

Consider the system (31), our goal is to find decentralized control law $u = Ky_x$ that minimize the cost function

$$J^* = \int_{-\infty}^{+\infty} (X^T QX + u^T Ru)dt \quad (32)$$

such that the closed loop system

$$S_c = \begin{cases} \dot{X} = (E(t_i)^{-1} A(t_i) + E(t_i)B(t_i)KC(t_i))X \\ y = C(t_i)X \end{cases} \quad (33)$$

is asymptotically stable, where the values of Q and R are chosen using trial error method.

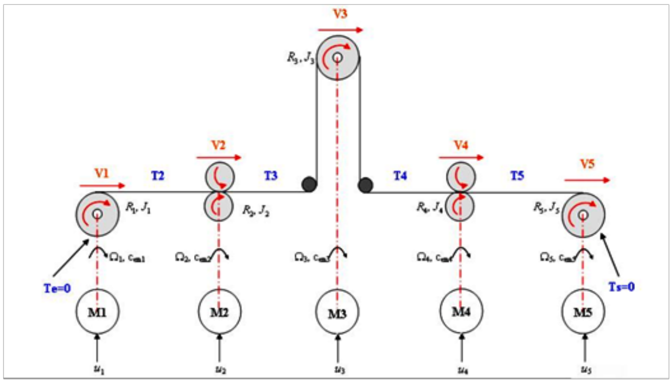

Fig.2 Scheme of five motors web winding system [8]

B. *Problem solution*

Since the goal of W.W.S is to maximize the output velocity V_3 and at the same time control the output tensions T_u, T_w, we generate an optimal output feedback of the form $u_i = K_i y_i ; i = 1,2$ for each subsystem, and then we associate the performance index

$$J_i^* = \int_{-\infty}^{+\infty} (X_i^T Q_i X_i + u_i^T R_i u_i)dt \qquad i = 1,2 \tag{34}$$

The necessary and sufficient conditions of optimality for each subsystem are [18]

$$\begin{cases} \phi_i^T P_i + P_i \phi_i + Q_i + C_{xi}^T K_i^T R_i K_i C_{xi} = 0 \\ K_i = -R_i^{-1} B_{xi}^T P_i L_i C_{xi}^T (C_{xi} L_i C_{xi}^T)^{-1} \\ \phi_i L_i + L_i \phi_i^T + X_{0i} = 0 \end{cases} \tag{35}$$

Where $\phi_i = (E(t_i)A)_i + (E(t_i)B)_i K_i C_{xi} \quad X_{0i} = x_{0i} x_{0i}^T$; generally, we take $x_{0i} = I$

Equation (35) can be solved in three steps, first we must find the value of L_i, this value allows us to calculate the gain matrix K_i from the second equation of (35), then by solving the first equation in (35) we find P_i that allows us to find J_i and compare it to the desired J_i^*, if the error isn't small enough the process will be repeated with the calculated state values x_i. If we assume that the extended system is stabilizable under the closed loop structure, the optimal cost for the expanded problem is

$$J_e^*(x_{e0}) = trace(P_e X_{e0})$$

The necessary conditions for K_i to be an optimal solution is

$$\frac{\partial}{\partial K_i} = tra\,ce\left[L_i P_i B_i K_i C_i + \frac{1}{2} L_i C_i^T K_i^T R_i K_i C_i \right] = 0$$

Since in many applications $X(0)$may not be known, we usually do not minimize the P.I but its expected value is

$$E\left\{J_i^*\right\} = \frac{1}{2} E\left\{x_i^T(0) P x_i(0)\right\} = \frac{1}{2} tra\,ce(P_i X_{0i}) \tag{36}$$

and the optimal control law is found as $u_i = K_i y_i$,where

$$K_i = \begin{bmatrix} K_{11}^i & K_{12}^i & K_{13}^i & K_{14}^i \\ K_{21}^i & K_{22}^i & K_{23}^i & K_{24}^i \end{bmatrix} \tag{37}$$

By applying conditions in eq (36), the control law for expanded system is written a

$$K_i = \left[\begin{array}{cccc|cccc} K_{11}^1 & K_{12}^1 & 0 & 0 & K_{13}^1 & K_{14}^1 & 0 & 0 \\ K_{21}^1 & K_{22}^1 & 0 & 0 & K_{23}^1 & K_{24}^1 & 0 & 0 \\ \hline 0 & 0 & K_{11}^2 & K_{12}^2 & 0 & 0 & K_{13}^2 & K_{14}^2 \\ 0 & 0 & K_{21}^2 & K_{22}^2 & 0 & 0 & K_{23}^2 & K_{24}^2 \end{array}\right] \tag{37}$$

and the contracted control law for the original system

$$K = \left[\begin{array}{c|c|c|c|c|c} K_{11}^1 & K_{12}^1 & 0 & K_{13}^1 & K_{14}^1 & 0 \\ \hline K_{21}^1 & K_{22}^1 + K_{11}^2 & K_{12}^2 & K_{21}^1 & K_{24}^1 + K_{13}^2 & K_{14}^2 \\ \hline 0 & K_{21}^2 & K_{22}^2 & 0 & K_{23}^2 & K_{24}^2 \end{array}\right] \tag{38}$$

This controller is considered as fully decentralized overlapped controller for the overlapped system. To apply this control law for the original system, we must write it in the form

$$u = Fy + Lv + w \tag{39}$$

where w is the external input applied to the three motors web winding system [20].

We have

$$\begin{cases} K = [F \;\; L] \\ K_e = [F_e \;\; L_e] \end{cases} \tag{40}$$

With

$$\begin{cases} F = \left[\begin{array}{c|c|c} K_{11}^1 & K_{12}^1 & 0 \\ \hline K_{21}^1 & K_{22}^1 + K_{11}^2 & K_{12}^2 \\ \hline 0 & K_{21}^2 & K_{22}^2 \end{array}\right] \\ L = \left[\begin{array}{c|c|c} K_{13}^1 & K_{14}^1 & 0 \\ \hline K_{23}^1 & K_{24}^1 + K_{13}^2 & K_{14}^2 \\ \hline 0 & K_{23}^2 & K_{24}^2 \end{array}\right] \end{cases} \tag{41}$$

Now, the projection of this control law onto the original system gives

$$\begin{cases} \dot{X} = (E(t_i)^{-1} A(t_i) + E(t_i)^{-1} B(t_i) K C(t_i)) X \\ y = C(t_i) X \end{cases} \tag{42}$$

VI. Results and Discussion

In this part, we give results of simulation of five motor web winding system control using proposed controller (green) and for H infinity controller (red).

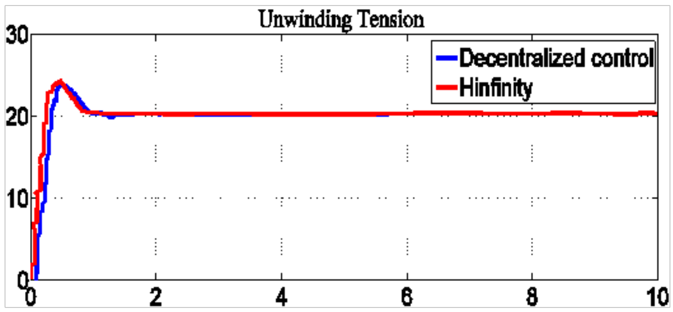

Fig.3.Web tension for the proposed (blue) and H infinity controllers (red) – Unwinding motor

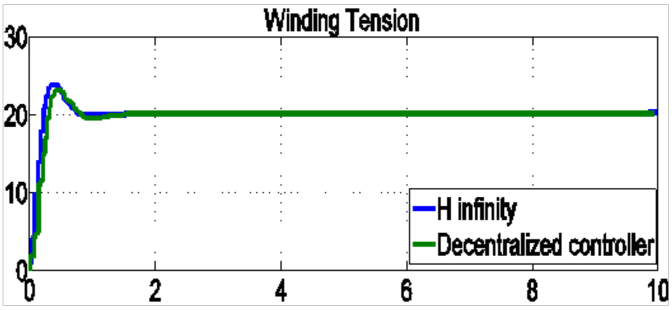

Fig.4.Web tension for the proposed (green) and H infinity controllers (blue) – Winding motor

In order to set the advantages of overlapping decomposition technique we have compared the results with H infinity technique recently developed in [20]. As shown in Figures 3 and 4, the desired output feedback control signals for web tension are reached, where

- For H infinity, the controller tracks the input tension with errors $e_1 = 11.4\%$ (Fig.3) and $e_2 = 7.6\%$ (Fig.4), where the cost functions are calculated from equation (36) $J^*_{01} = 3.21\times10^3$ and $J^*_{02} = 1.9\times10^3$.
- For the proposed algorithm, the controller is perfectly follows the desired input tensions, where the calculated errors are $e_{1d} = 3.5\%$ (Fig.3) and $e_{2d} = 4.1\%$ (Fig.4) the cost function values are equal to $J^*_{1d} = 2.5\times10^2$ and $J^*_{2d} = 1.9\times10^2$.

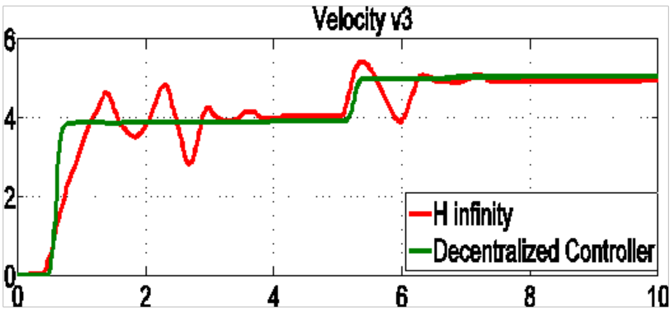

Fig.5. Speed tension for the proposed and H infinity controllers – Motor3

As shown in Fig. 5, the desired output feedback control signal for web speed is also reached, where

- For H infinity: the controller follows the input speed signal with errors $e_1 = 10.7\%$ (Fig.5) and $e_2 = 12.9\%$ (Fig.5) where the cost function values are equal to $J^*_{11} = 5.1\times10^3$ and $J^*_{12} = 6.3\times10^3$.
- For the proposed algorithm, the controller is perfectly follows the desired input signal where the calculated error is $e_{1d} = 7.5\%$ (Fig.5) and $e_{2d} = 3.8\%$ (Fig.5) where the cost values are $J^*_{1d} = 1.38\times10^2$ and $J^*_{2d} = 5.61\times10^2$.

Using the comparative study given in Tab.1, We can extract from the last column that the proposed algorithm is more robust and more effective in this type of systems, the values in this column is calculated by

$$camp = \frac{\text{H infinity - proposed controller}}{H \text{ infinity}}\% \quad (30)$$

Table I

A Comparative Study of H Infinity and the Proposed Controller

			H infinity	**Proposed algorithm**	**Camp (%)**
Subsystem 1	**Tu**	Error (%)	11.4%	3.5%	51.33%
		Cost value	3.21×10^3	2.5×10^2	93.2%
	V3	Error (%)	10.7%	7.5%	11.22%
		Cost value	5.1×10^3	1.38×10^2	94.0%
Subsystem 1	**T w**	Error (%)	7.6%	4.1%	53.66%
		Cost value	1.9×10^3	1.9×10^2	92.7%
	V3	Error (%)	12.9%	3.8%	31%
		Cost value	6.3×10^3	5.61×10^2	90.84%

To validate these results, we have used other type of input signals as it is showed in figure 7 and 8.

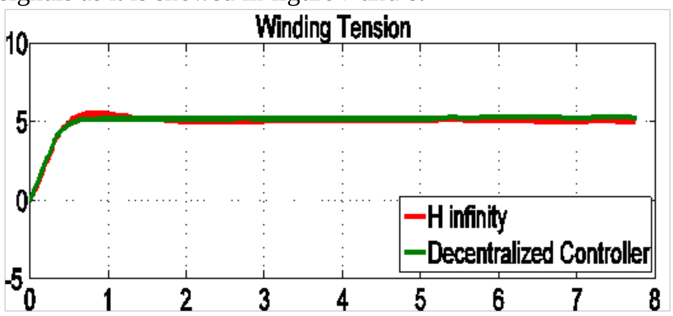

Fig.7. Winding tension response for second input signal

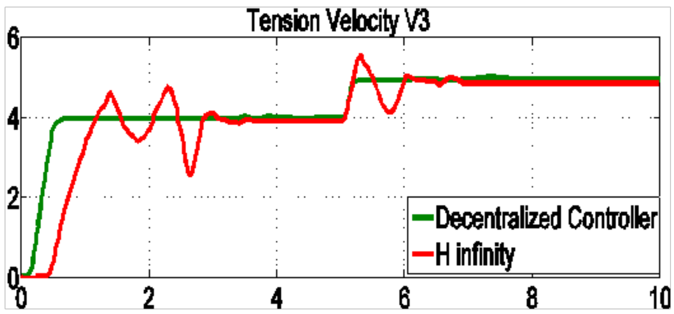

Fig.8. Velocity response for second input signal

- For H infinity controller: unwinding motor is stabilized in $t = 1s$ with value of 5N (Fig7), the value of the cost function is $J_1^* = 5.65\times10^3$.
- For proposed controller: winding motor response is stabilized in 500ms with peak of $5N$ (Fig.7), The value of the cost function is $J_2^* = 823.58$.

In comparing with optimal centralized control which gave cost function of $J_t^* = 1.10 \times 10^4$, it is clear that $J_1^* + J_2^* \prec J_t^*$; which means that this decomposition approach doesn't just minimize complexity of calculation but also minimize the value of cost function for the optimal system. We notice here that time response of optimal system is less than the time in non-optimal system.

VII. CONCLUSIONS

A longitudinal topology of multi-overlapping decomposition strategy has been proposed in this paper as control design for industrial web winding system; the mathematical model of web winding system has been introduced to provide mathematical condition of decomposition. Optimal decentralized dynamic output feedback controllers design has been proposed for five motors web winding system with multi-overlapping longitudinal topology. We found that multi-overlapping decentralized controller gives better results in comparison to H infinity controller, in addition to simplification and reducing complexity. Furthermore to improve these results we have developed an optimization technique that allows us not just design optimal controller but also minimize the value of the cost function of the whole system in comparison to original one. To highlight the advantages of the proposed algorithm, a comparison between the algorithm given in this paper and H infinity algorithm is sited at the end of this paper. Several assumptions made in this paper may be questioned for further study; larger systems may be studied, especially with other topology or combination of different topologies.

REFERENCES

[1] X. Li, H. Guo, and X-B. Chen " Robust overlapping decentralized control for multi-area power system with radial structure," IEEE conference of Intelligent Control and Automation, vol. 1, pp.1207-1211 , 2006.

[2] K. C. Lin, "Observer based tension feedback control with friction and inertia compensation," IEEE TRANS CONTROL SYST TECHN, vol.11, No.1, pp.109-118 , 2003.

[3] P. Q. Francisco, J. Rodellar, and J.M. Rosell, " Sequential design of multi-overlapping controllers for longitudinal multi-overlapping systems," Applied mathematics and computation, vol. 207, No. 3, pp.1170-1183 , 2010.

[4] L. Iftar, " Decentralized optimal control with overlapping decompositions," IEEE international conference on system engineering, Dayton. OH.USA, pp. 299-302, 1991.

[5] A. Iftar, E.J. Davison " Decentralized control strategies for dynamic routing," Journal of Optimal Control Applications and Methods, vol. 23, pp. 329-335 , 2002.

[6] A. Iftar, U. Ozguner, " Overlapping decompositions, expansions, contractions, and stability of Hybrid systems," IEEE Trans. Automatic Control , Vol. 43, No. 8, pp.1040-1055, 1998.

[7] M. Ikeda, D.D. Siljack, and D.E. white, " Decentralized control with overlapping information sets," Journal of Optimization theory and applications, Vol. 34, pp. 279-310, 1981.

[8] L. Bakule, J. Rodellar, and J.M. Rosell, " Structure of expansion contraction matrices in the inclusion principle for dynamic systems," SIAM J. Matrix Anal. Appl., Vol.21, No.4, pp. 1136-1155, 2000.

[9] M. Ikeda, D.D. Siljack, " Overlapping decentralized control with input, state, and output inclusion," Journal Control theory and advanced technology, Vol. 2, No .2, pp.155-172, 1986.

[10] Y.H. Jung, J.W. Choi, and Y.B. Seo, " Overlapping decentralized EA control Design for an active Suspension system of a Full Car Model," SICE 2000, Lizuka, July 26-28, 2000.

[11] A. N. GÜÇLÜ, A. B. ÖZGÜLER, " Diagonal stabilization of linear multivariable systems," International Journal of Control, vol. 43, No.3, pp. 965–980, 1986.

[12] M. Kidouche, H. Habbi, and M. Zelmat, " Stability of Interconnected Systems under Structural Perturbation: Decomposition- Aggregation Approach," World Academy of Science, Engineering and Technology Vol. 37, 2008.

[13] M. Mahmoud, M. Hassen, and M. Darwish, " Large-Scale Control Systems: Theories and Techniques," Marcel-Dekker, New York, 1985.

[14] D. Whiteside, " Basics of web tension control summary," Tension Control Maxcess International, PLACE conference, st Louis, MO,2007.

[15] D.Siljak, " Large-Scale Dynamic Systems: Stability and Structure," North Holland, Amsterdam, 1978,.

[16] M. E. Khatir, E. J. Davison, " Decentralized control of a large platoon of vehicles using non-identical controllers," Proc. of the 2004 American Control Conference, pp. 2769-2776, Boston, 2004.

[17] V. A. Ugrinovski, I. R. Petersen, A. U. Sarkin, and E. Y. Ugrinoskaya, " Decentralized state feedback stabilization and robust of uncertain large scale systems with integrally constrained interconnections," Systems & Control Letters, Vol. 40, pp. 107–119, 2000.

[18] F. L. Chernovski, X. G. Yan, J. Lam, H. S. Li, and I. M. Chen, " Decentralized control of nonlinear large-scale systems using dynamic output feedback," Journal of Optimization Theory and Applications, Vol. 104, No. 2, pp.459–475 ,2000.

[19] A.I. Zecevic, and D.D. Šiljak, " A new approach to control design with overlapping information structure constraints," Automatica, Vol. 41, pp. 265 – 272 , 2005.

[20] W. Zhou, " Robust and decentralized control of web winding systems," *Ph.D Thesis*, Cleveland State University, November 2007.